# Systems of Equations - Substitution

**Objective: Solve systems of equations using substitution.**

When solving a system by graphing has several limitations. First, it requires the graph to be perfectly drawn, if the lines are not straight we may arrive at the wrong answer. Second, graphing is not a great method to use if the answer is really large, over 100 for example, or if the answer is a decimal the that graph will not help us find, 3.2134 for example. For these reasons we will rarely use graphing to solve our systems. Instead, an algebraic approach will be used.

The first algebraic approach is called substitution. We will build the concepts of substitution through several example, then end with a five-step process to solve problems using this method.

**Example 1.**

| | |
|---|---|
| $x = 5$<br>$y = 2x - 3$ | We already know $x = 5$, substitute this into the other equation |
| $y = 2(\mathbf{5}) - 3$ | Evaluate, multiply first |
| $y = 10 - 3$ | Subtract |
| $y = 7$ | We now also have $y$ |
| $(5, 7)$ | Our Solution |

When we know what one variable equals we can plug that value (or expression) in for the variable in the other equation. It is very important that when we substitute, the substituted value goes in parenthesis. The reason for this is shown in the next example.

**Example 2.**

| | |
|---|---|
| $2x - 3y = 7$<br>$y = 3x - 7$ | We know $y = 3x - 7$, substitute this into the other equation |
| $2x - 3(\mathbf{3x - 7}) = 7$ | Solve this equation, distributing $-3$ first |

| | |
|---|---|
| $2x - 9x + 21 = 7$ | Combine like terms $2x - 9x$ |
| $-7x + 21 = 7$ | Subtract 21 |
| $\underline{\mathbf{-21 \quad -21}}$ | |
| $-7x = -14$ | Divide by $-7$ |
| $\overline{\mathbf{-7}} \quad \overline{\mathbf{-7}}$ | |
| $x = 2$ | We now have our $x$, plug into the $y =$ equation to find $y$ |
| $y = 3(\mathbf{2}) - 7$ | Evaluate, multiply first |
| $y = 6 - 7$ | Subtract |
| $y = -1$ | We now also have $y$ |
| $(2, -1)$ | Our Solution |

By using the entire expression $3x - 7$ to replace $y$ in the other equation we were able to reduce the system to a single linear equation which we can easily solve for our first variable. However, the lone variable (a variable without a coefficient) is not always alone on one side of the equation. If this happens we can isolate it by solving for the lone variable.

**Example 3.**

| | |
|---|---|
| $3x + 2y = 1$ <br> $\boldsymbol{x} - 5y = 6$ | Lone variable is $x$, isolate by adding $5y$ to both sides. |
| $\underline{\mathbf{+5y + 5y}}$ | |
| $x = 6 + 5y$ | Substitute this into the untouched equation |
| $3(\mathbf{6 + 5y}) + 2y = 1$ | Solve this equation, distributing 3 first |
| $18 + 15y + 2y = 1$ | Combine like terms $15y + 2y$ |
| $18 + 17y = 1$ | Subtract 18 from both sides |
| $\underline{\mathbf{-18 \qquad -18}}$ | |
| $17y = -17$ | Divide both sides by 17 |
| $\overline{\mathbf{17}} \quad \overline{\mathbf{17}}$ | |
| $y = -1$ | We have our $y$, plug this into the $x =$ equation to find $x$ |
| $x = 6 + 5(\mathbf{-1})$ | Evaluate, multiply first |
| $x = 6 - 5$ | Subtract |
| $x = 1$ | We now also have $x$ |
| $(1, -1)$ | Our Solution |

The process in the previous example is how we will solve problems using substitu-

# 4.2 Practice - Substitution

**Solve each system by substitution.**

1) $y = -3x$
$y = 6x - 9$

2) $y = x + 5$
$y = -2x - 4$

3) $y = -2x - 9$
$y = 2x - 1$

4) $y = -6x + 3$
$y = 6x + 3$

5) $y = 6x + 4$
$y = -3x - 5$

6) $y = 3x + 13$
$y = -2x - 22$

7) $y = 3x + 2$
$y = -3x + 8$

8) $y = -2x - 9$
$y = -5x - 21$

9) $y = 2x - 3$
$y = -2x + 9$

10) $y = 7x - 24$
$y = -3x + 16$

11) $y = 6x - 6$
$-3x - 3y = -24$

12) $-x + 3y = 12$
$y = 6x + 21$

13) $y = -6$
$3x - 6y = 30$

14) $6x - 4y = -8$
$y = -6x + 2$

15) $y = -5$
$3x + 4y = -17$

16) $7x + 2y = -7$
$y = 5x + 5$

17) $-2x + 2y = 18$
$y = 7x + 15$

18) $y = x + 4$
$3x - 4y = -19$

19) $y = -8x + 19$
$-x + 6y = 16$

20) $y = -2x + 8$
$-7x - 6y = -8$

21) $7x - 2y = -7$
$y = 7$

22) $x - 2y = -13$
$4x + 2y = 18$

23) $x - 5y = 7$
$2x + 7y = -20$

24) $3x - 4y = 15$
$7x + y = 4$

25) $-2x - y = -5$
$x - 8y = -23$

26) $6x + 4y = 16$
$-2x + y = -3$

27) $-6x + y = 20$
$-3x - 3y = -18$

28) $7x + 5y = -13$
$x - 4y = -16$

29) $3x + y = 9$
$2x + 8y = -16$

30) $-5x - 5y = -20$
$-2x + y = 7$